Worksheet for September 3

Problems marked with an asterisk are to be placed in your math diary.

(1*.) Recall from Calculus 1 that, using L'Hospital's Rule, we have $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. Can you use this to justify the equation $\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{x+y} = 1$? If so, explain why.

2. Calculate the limits, or justify why the limits do not exist.

(i) $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$. (Assume $x \neq y$.)

(ii) $\lim_{(x,y)\to(2,0)} \frac{x^2-y^2-4x+4}{x^2+y^2-4x+4}$.

(iii) $\lim_{(x,y,z)\to(0,0,0)} \frac{2x^2+3y^2+z^2}{x^2+y^2+z^2}$.

(3*). At what points in $\mathbb{R}^2$ is the function $f(x,y) = \begin{cases} \frac{x^3+x^2+xy^2+y^2}{x^2+y^2}, \text{ if } (x,y) \neq (0,0) \\ 2, \text{ if } (x,y) = (0,0) \end{cases}$ continuous?

(4.*) Suppose $F(x,y) = (5x+2, 2y+1)$. Use an epsilon-delta argument to show $\lim_{(x,y)\to(1,1)} F(x,y) = (7,3)$.